Method f Determining Implied Volatility for American Options

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Related Applications

This application claims the benefit of U.S. Provisional Application Serial No. 60/422,231 filed October 30, 2002, which is fully incorporated herein by reference.

Field of the Invention

The present invention relates to a new method for determination of a financial index, namely, *implied volatility* for American options.

Background of the Invention

Implied volatility is a quantity related to the price of a traded financial option to buy ("call" option) or to sell ("put" option) a particular asset. Specifically, implied volatility represents the market's estimate of the future price volatility of the underlying asset. According to results of Black and Scholes, the market price of an option depends solely on the asset's future price volatility, the current price of the underlying asset, the risk-free interest rate, the dividend yield (if any) from the underlying asset, the exercise price of the option, and the time to expiration.

In the case of "European" options (options which can only be exercised at expiration), Black and Scholes present a formula for the calculation of the fair market price of the option given its volatility and other inputs. Subsequent researchers showed

how this formula could be inverted, essentially solving for the volatility using the other inputs and the current market price of the option. This value is referred to as implied volatility, and is used by options traders as an indication of the relative value of an option, in much the same way that yield serves as a measure of the relative value of a bond. An equivalent Black-Scholes type formula does not exist for American options.

Option pricing and implied volatility are known in the art, and further discussion of them provided in the following publications, all of which are fully incorporated herein by reference: Black, F. and M. Scholes, 1973, "The Pricing of Options and Corporate Liabilities", Journal of Political Economy, 81, 637-654; Cox, J. C., S. A. Ross, and M. Rubinstein, 1979, "Option Pricing: ASimplified Approach", Journal of Financial Economics, 7, 229-263; Cox, J. C., and M. Rubinstein, 1985, "Options Markets", Prentice-Hall, Englewood Cliffs, NJ; and Latane, H., and R. Rendleman, Jr., 1976, "Standard Deviation of Stock Price Ratios Implied in Option Prices", Journal of Finance, 31, 369-382.

Summary of the Invention

It is an object of the present invention to provide a new method for use in determining implied volatility.

Further to the invention, a new method is provided for determining vega. Preferably, the method is computer-implemented, using any desired computing device, whether a desktop, notebook, handheld computer, or so forth.

Brief Description of the Figures

Figure 1 sets forth a method for calculation of the node vega in accordance with the present invention, wherein equation (5) shows calculation of the "node vega" if no early exercise occurs at the node, and equation (6) sets forth the node vega if early exercise occurs at the node.

Detailed Description of the Invention and the Preferred Embodiments

As is known in the art, an "American" option is an option which can be exercised prior to expiration. As noted above, an equivalent Black-Scholes type formula does not exist for American options. Instead, they are generally priced using a industry-standard Cox-Ross-Rubinstein (CRR) binomial tree model. This model can accommodate underlying securities with either discrete dividend payments or a continuous dividend yield.

In the framework of the CRR model, the time between now and option expiration is divided into N sub-periods. Over the course of each sub-period, the security price is assumed to move either "up" or "down". The size of the security price move is determined by the volatility and the size of the sub-period. Specifically, the security price at the end of sub-period i is given by one of the following:

Equation (1):

$$S_{i+1}^{up} = S_i u \equiv S_i \exp(\sigma\sqrt{h})$$

Equation (2):

$$S_{i+1}^{down} = S_i d \equiv S_i \exp(-\sigma\sqrt{h})$$

where $h \equiv T/N$ is the size of the sub-period, and S_i is the security price at the beginning of the sub-period, S_{i+1}^{up} is the security price in the event of an up move over the sub-period i, and S_{i+1}^{down} is the security price in the event of an down movement over the sub-period i, and σ is the volatility.

The price of a call option at the beginning of each sub-period is dependent on its price at the end of the sub-period, and is given by:

Equation (3):

$$C_i = \max \left\{ \begin{array}{c} \left[p C_{i+1}^{up} + (1-p) C_{i+1}^{down}\right]/R \\ S_i - K \end{array} \right\}$$

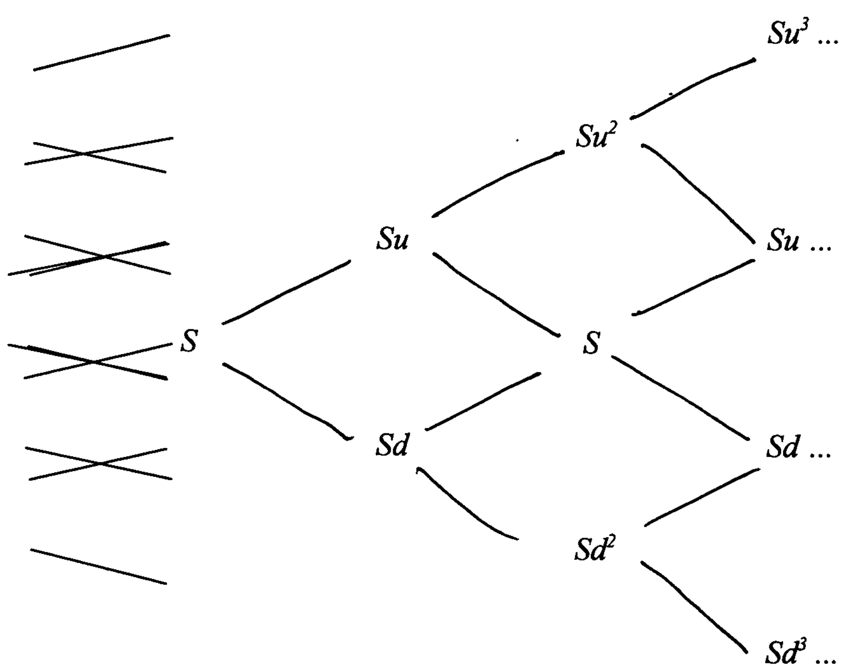
and likewise for a put option. Here, r is the interest rate, q is the continuous dividend yield (if the security is an index), $R \equiv exp([r\text{-}q]h)$, and C_{i+1}^{up} and C_{i+1}^{down} are the price of the option at the end of the sub-period, depending on whether the security price moves "up" or "down", and K is the option strike price. The "risk-neutral" probability p is given by:

Equation (4):

$$p = \frac{R - d}{u - d}$$

where u and d are as defined above in Equations (1) and (2).

To use the CRR approach to value an option, we start at the current security price S and build a "tree" of all the possible security prices at the end of each sub-period, under the assumption that the security price can move only either up or down:



The tree is constructed out to time T (option expiration).

Next, the option is priced at expiration by setting the option expiration value equal to the exercise value: $C = \max(S_T - K, 0)$ and $P = \max(K - S_T, 0)$, where S_T is the stock value at time T (option expiration), and dependent on the node. The option price at the beginning of each sub-period is determined by the option prices at the end of the sub-period, using the formula above. Working backwards, the calculated price of the option at time $i=0$ is the theoretical model price.

The CRR model is adapted to securities that pay discrete dividends as follows: When calculating the price of the option from equation (1), the security price S_i used in the equation is set equal to the original tree price S_i^0 *minus* the sum of all dividend payments received between the start of the tree and time i. Under the constant dividend yield assumption, this means that the security price S_i used in equation (1) should be set equal to $S_i^0(1-n\delta)$, where S_i^0 is the original tree price, δ is the dividend yield, and n is the number of dividend payments received up to time i. All other calculations are the same.

The CRR model usually requires a very large number of sub-periods to achieve good results (typically, $N > 1000$), and this often results in a large computational requirement.

To compute the implied volatility of an option given its price, a model such as CRR is run iteratively with new values of σ until the model price of the option converges to its market price, defined as the midpoint of the option's best closing bid and best closing offer prices. At this point, the final value of σ is the option's implied volatility. A numerical optimization technique such as Newton-Raphson is typically used for this calculation. For Newton-Raphson as well as other optimization algorithms, an important input is the option's *vega* (sometimes called *kappa*), defined as the partial derivative of

the option's price with respect to the volatility. The vega cannot be calculated directly from the CRR model however, so it must be approximated by "tweaking" the model. In this approach, the CRR model is run twice, once using the current value of volatility and once using this same value of volatility increased by a small amount (the "tweak"). Thus for each iteration two values of the option price are calculated. The difference between these two values divided by the amount of the "tweak" is approximately equal to the vega of the option. This vega value, as well as the current volatility estimate and the calculated option price, are fed into the optimization algorithm, and a new volatility estimate is generated; this loop continues until the calculated option price equals the targeted market price.

Because of the "tweaking", each iteration in the calculation of implied volatility typically requires two separate CRR calculations in the method of the prior art.

In accordance with the present invention, an improved method is provided for determining vega. Preferably, the method is computer-implemented, with vega being calculated using any desired computing device, whether a desktop, notebook, handheld computer, or so forth.

In the method of the invention, at each node of the CRR tree, two values are calculated: C_i, the option price at that stage of the tree, and V_i, the node vega, which is the exact derivative of the option price with respect to the volatility at that particular stage of the tree. This approach allows the vega to be calculated at the same time that the option price is calculated. Although more calculation is required at each node, the whole calculation takes significantly less time than running the entire CRR model twice for each iteration. In addition, the calculated vega is exactly equal to the derivative of the

CRR option price with respect to the volatility, rather than an approximation from "tweaking", so the implied volatility optimization algorithm has more exact inputs and thus converges faster.

The "node vega" is calculated according to the expression in Equation (5) (assuming that no early exercise occurs at the node) of Figure 1, where C^{up}_{i+1} is the "up" node option price at the end of the subperiod, C^{down}_{i+1} is the "down" node option price at the end of the subperiod, V^{up}_{i+1} and V^{down}_{i+1} are the up and down vegas at the end of the subperiod, and p is the "risk neutral probability". If early exercise occurs at the node, the vega at the node is given by Equation (6), where S is the stock price at the node, where the stock price assigned to a particular node is indirectly a function of the volatility. This approach allows the vega to be calculated at the same time that the option price is calculated.

If desired, the implied volatility can be computed using this method for both puts and calls across a range of different strike prices. Experience has shown that the implied volatility commonly varies across this range. Likewise, the present method can be used to plot the "volatility smile" (a graph of the implied volatility against the strike price), and can also be used as an improved method to determined the presence of a "volatility skew" (a difference between the implied volatilities using equal out of the money calls and puts). Some options trading is based on interpretation of such skew, and can be conducted more effectively using the method of the present invention. For example, high skew ratios can indicate that demand is increasing for puts, and so forth. In accordance with the invention, the computing device and associated software can provide data on the volatility smile, the volatility skew, and likewise with any other desired tool for analysis used by those in the art of options trading.

In further embodiments of the invention, the present new method of calculating vega is used in other options related applications as well. Thus, it applies not only to the CRR model, but to all tree-based and grid-based models of calculating option prices. Likewise, the calculations set forth above can be conducted at any point in a sub-period consistent with the invention, whether at the beginning or end or a sub-period.

Having described this invention with regard to specific embodiments, it is to be understood that the description is not meant as a limitation since further embodiments, modifications and variations may be apparent or may suggest themselves to those skilled in the art. It is intended that the present application cover all such embodiments, modifications and variations.